Kinetic approach to low-frequency waves in dusty self-gravitating plasmas

Victoria V. Yaroshenko,* Gerald Jacobs, and Frank Verheest
Sterrenkundig Observatorium, Universiteit Gent, Krijgslaan 281, B-9000 Gent, Belgium
(Received 17 November 2000; published 29 May 2001)

A kinetic model is derived for the propagation of low-frequency waves in a dusty plasma containing very heavy dust particles, when the self-gravitational interaction due to these grains is included in the analysis. Analytical expressions for the dispersion function are used to examine the instability and damping of the modes. The stability regions of low-frequency waves are compared in the kinetic and the analogous hydrodynamic models, showing that there are only slight differences. However, the kinetic analysis modifies the growth rates of the Jeans instability and can considerably alter the conditions for the propagation of stable dust modes.

DOI: 10.1103/PhysRevE.63.066406 PACS number(s): 52.27.Lw, 52.35.Fp, 95.30.Qd

I. INTRODUCTION

Dust particles immersed in plasma and radiative environments inevitably become electrically charged. Such mixtures of electrons, ions, and charged dust grains distinguish themselves from ordinary plasmas, because charged dust grains are much more than just an additional ionic species. One of the important novel features of dusty plasmas when compared with the usual multispecies electron-ion plasmas, is that the dust particles can interact through both electric and gravitational forces. When self-gravitational interactions due to the heavier dust component are included, dusty plasmas are subject to a Jeans instability. This results in a significant modification of collective modes and in new stability conditions.

The first and one of the most studied low-frequency modes in dusty plasmas is the ''dust-acoustic wave'' [1], which has been confirmed in recent laboratory experiments [2,3]. On the other hand, when self-gravitational effects are included self-consistently, new stable and unstable modifications of this mode in self-gravitating plasmas are found. In particular, several authors have discussed new conditions for the existence of dust-acoustic waves in self-gravitating plasmas [4–9]. For a further discussion of wave processes in such a medium, in general, we refer to recent books [10,11].

Most work on low-frequency modes in dusty and self-gravitating plasmas was based on the hydrodynamic approach that, generally speaking, breaks down at phase velocities small compared to the thermal velocities of the particles and is clearly insufficient to describe the thermal influences on different kinds of waves. The most general model to study the effects associated with the thermal motion of particles is the kinetic description, which is based on the statistical representation of the medium as a system of a large number of particles.

The most important kinetic effect in plasmas is the collisionless dissipation of wave energy, known as Landau damping, which is well studied for different types of waves in a conventional electron-ion plasma [12,13]. Recently, a kinetic model for the propagation of dust-acoustic modes in a dusty plasma was treated [14,15], where the effect of dust size distributions on the propagation and damping of the low-frequency waves was considered.

The kinetic description of a neutral gravitating medium leads to some new phenomena compared to the fluid model. In both cases there is instability if the wave number is smaller than the Jeans wave number. However, the oscillations in these two models are quite different: the fluid model supports short-wavelength sound waves and the kinetic one does not. The latter results in a strong damping effect for short-wavelength sound perturbations [16,17].

Since the dust particles of a self-gravitating plasma interact through both electric and gravitational forces, a kinetic model for the collective effects will eventually account for the specific features of a plasma and a neutral gravitating medium. Hence it is physically relevant to investigate low-frequency modes in such a complex plasma system in a kinetic description.

In the present paper we study the peculiarities of the propagation of eigenmodes in self-gravitating plasmas, and more specifically, the growth and damping rates of the instabilities of dust-acoustic modes and dust Langmuir modes. Besides purely kinetic effects such as Landau damping, there is another kind of damping mechanism for the dust modes, namely, the charge-fluctuation damping due to variable dust charges [18,19]. The first analysis of the charge-fluctuation problem in dense self-gravitating plasmas has been given by Rao and Verheest [20], and shows that there are order-of-magnitude differences between the characteristic times of the wave processes on the one hand and the charging times on the other. The charging times are short enough and hence dust grains have sufficient time to achieve an average charge on the collapse timescales. Therefore, when considering a low-frequency regime in a self-gravitating plasma, we can assume that grains have constant charges, thus omitting for simplicity a very weak damping due to charge variations.

The plan of the paper is as follows. In Sec. II we derive the general kinetic dispersion relation and consider the unstable solutions (Jeans-like perturbations). The plasma modes are analyzed in Sec. III for various frequency regimes. Finally, brief conclusions are given in Sec. IV.

*Permanent address: Institute of Radio Astronomy of National Academy of Science of Ukraine, Chervonopraporna 4, Kharkov 310002, Ukraine.

II. KINETIC MODEL OF A DUSTY SELF-GRAVITATING PLASMA

A. Kinetic equation

We consider low-frequency plasma waves in infinite, homogeneous, unmagnetized, and collisionless self-gravitating plasmas, which consist of warm electrons (with subscript e), ions (with subscript i), and heavy charged grains of a single sort (with subscript d). All particle species will be described by a distribution function f_α (with $\alpha = e,i,d$) in phase space that satisfies the ordinary Vlasov equation

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{\nabla}\cdot(\mathbf{v} f_\alpha) + \boldsymbol{\nabla}_v \cdot \left[f_\alpha \left(\frac{q_\alpha}{m_\alpha}\mathbf{E} - \boldsymbol{\nabla}\phi \right) \right] = 0. \qquad (1)$$

Here q_α and m_α denote the particle charges and masses, respectively, $\boldsymbol{E}$ denotes the electric field, and ϕ denotes the gravitational potential.

The self-consistent electric and gravitational fields can be found from the Poisson equations

$$\nabla^2 \varphi = -\frac{1}{\varepsilon_0} \sum_\alpha q_\alpha n_\alpha , \qquad (2)$$

$$\nabla^2 \phi = 4\pi G \sum_\alpha m_\alpha n_\alpha , \qquad (3)$$

where φ is the electric potential, G is the gravitational constant, and

$$n_\alpha = \int f_\alpha d^3\mathbf{v}. \qquad (4)$$

B. Dispersion relation

We now consider plasma waves, assuming all the external fields and average velocities of the particles to be zero in the unperturbed (equilibrium) state. Following the standard linearization procedure we obtain the perturbed distribution functions

$$f_{\alpha 1} = -\frac{\mathbf{k}\cdot\boldsymbol{\nabla}_v f_{\alpha 0}}{\omega - \mathbf{k}\cdot\mathbf{v}} \left[\frac{q_\alpha}{m_\alpha}\varphi + \phi \right]. \qquad (5)$$

Here the values with subscript 0 refer to the zeroth-order state, whereas first-order terms are indicated by a subscript 1 and are assumed to vary as $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$.

Substituting the perturbed distribution function (5) into the Poisson equations (2) and (3), we obtain two coupled equations with respect to the electric and gravitational potentials, viz.,

$$\varphi \varepsilon_p + \phi \frac{K}{\sqrt{G}} = 0,$$
$$-\varphi K \sqrt{G} + \phi \varepsilon_G = 0. \qquad (6)$$

These involve a plasma dielectric constant

$$\varepsilon_p = 1 + \frac{1}{\varepsilon_0 k^2} \sum_\alpha \frac{q_\alpha^2}{m_\alpha} I_\alpha , \qquad (7)$$

its analog for a self-gravitating neutral medium

$$\varepsilon_G = 1 - \frac{4\pi G}{k^2} \sum_\alpha m_\alpha I_\alpha \qquad (8)$$

and a coupling factor

$$K = \sqrt{\frac{4\pi G}{\varepsilon_0}} \frac{1}{k^2} \sum_\alpha q_\alpha I_\alpha , \qquad (9)$$

where

$$I_\alpha = \int \frac{\mathbf{k}\cdot\boldsymbol{\nabla}_v f_{\alpha 0}}{\omega - \mathbf{k}\cdot\mathbf{v}} d^3\mathbf{v}. \qquad (10)$$

The dispersion relation for the electrostatic waves in the kinetic model of self-gravitating plasmas,

$$\varepsilon(\omega,k) = \varepsilon_p + \frac{K^2}{\varepsilon_G} = 0, \qquad (11)$$

thus has the same structure as in a fluid approach [21].

We now assume that the plasma particles are described by a Maxwellian distribution function

$$f_{\alpha 0} = \frac{n_{\alpha 0}}{(\pi v_{T\alpha}^2)^{3/2}} \exp\left(-\frac{v^2}{v_{T\alpha}^2} \right), \qquad (12)$$

where $n_{\alpha 0}$ is the equilibrium density, $v_{T\alpha} = (2k_B T_\alpha / m_\alpha)^{1/2}$ is the thermal velocity, and T_α is the temperature of the particles of type α. When $f_{\alpha 0}$ is of this form, the integral over all velocities in Eq. (10) can be calculated in rectangular coordinates (v_x, v_y, v_z), where the v_x axis is chosen to lie in the direction of $\mathbf{k}$. The integrals over v_y and v_z are simple, using $\int_{-\infty}^{\infty} \exp(-v^2/v_{T\alpha}^2) dv = \sqrt{\pi} v_{T\alpha}$, and Eq. (10) becomes

$$I_\alpha = \frac{2 n_{\alpha 0} k}{\sqrt{\pi} v_{T\alpha}^3} \int_{-\infty}^{\infty} \frac{v_x \exp(-v_x^2/v_{T\alpha}^2)}{k v_x - \omega} dv_x . \qquad (13)$$

C. Stable and unstable solutions

By analogy with the fluid case, we expect the boundary between stable and unstable solutions to occur at $\omega = 0$. At $\omega = 0$ the integral (13) is easily evaluated and we find

$$I_{\alpha,cr} = \frac{2 n_{\alpha 0}}{v_{T\alpha}^2}. \qquad (14)$$

Substituting Eq. (14) into Eqs. (7)–(9) and using Eq. (11), one can obtain the equation that determines the critical wave number k_{cr}, namely,

$$\left(1 + \frac{1}{k_{cr}^2 \lambda_D^2} \right) \left(1 - \frac{1}{k_{cr}^2 \lambda_{Jd}^2} \right) + \frac{1}{k_{cr}^2 \lambda_{Dd}^2} = 0. \qquad (15)$$

Here the characteristic Debye and Jeans lengths are given through $\lambda_{D\alpha}^2=\varepsilon_0 k_B T_\alpha/n_{\alpha 0}q_\alpha^2$ and $\lambda_{J\alpha}^2=k_B T_\alpha/4\pi G n_{\alpha 0}m_\alpha^2$, respectively. When considering waves in a self-gravitational plasma, it is absolutely meaningless to take gravitational interactions due to ions or electrons into account, which is indeed never done. Hence we neglect terms proportional to λ_{Je}^{-1} and λ_{Ji}^{-1}, which is equivalent to supposing that $|q_d|/m_d \ll |q_i|/m_i, e/m_e$. Electron and ion contributions are retained only via a global plasma Debye length λ_D, defined through $\lambda_D^{-2}=\lambda_{De}^{-2}+\lambda_{Di}^{-2}$.

The critical wave number may be thus easily determined as a positive root of the bi-quadratic equation (15), which we analyze below in two different regimes typical of low-frequency waves in dusty self-gravitating plasmas.

For the case of $k_{cr}^2\lambda_D^2 \ll 1$ (the dust-acoustic regime), the critical wave number is determined by the expression

$$k_{cr}^2=\frac{\omega_{Jd}^2}{v_{da}^2}\frac{1}{1+\lambda_{Dd}^2/\lambda_D^2}, \tag{16}$$

where $v_{da}=\omega_{pd}\lambda_D$ is the dust-acoustic velocity, $\omega_{p\alpha}=(n_{\alpha 0}q_\alpha^2/\varepsilon_0 m_\alpha)^{1/2}$ is the plasma frequency, and $\omega_{J\alpha}=(4\pi G n_{\alpha 0}m_\alpha)^{1/2}$ is the Jeans frequency. For cold dust, Eq. (16) simply reduces to the critical wave number k_{cr} for dust-acoustic waves in the fluid description.

In the regime of the dust Langmuir waves, when $k_{cr}^2\lambda_D^2 \gg 1$, k_{cr} is given by

$$k_{cr}^2=\frac{2(\omega_{Jd}^2-\omega_{pd}^2)}{v_{Td}^2}. \tag{17}$$

Pursuing the analogy with the fluid description of wave processes in self-gravitating plasmas, we suspect that all perturbations with wave numbers $k<k_{cr}$ will be unstable. To check this, we set $\omega=i\gamma$, where γ is real and positive and substitute this into Eq. (13). Using the relation

$$\int_0^\infty \frac{x^2\exp(-x^2)}{x^2+b^2}dx=\frac{1}{2}\{\sqrt{\pi}-\pi b\exp(b^2)[1-\mathrm{erf}(b)]\}, \tag{18}$$

where $\mathrm{erf}(b)$ denotes the error function, we find that

$$I_\alpha=\frac{2n_{\alpha 0}}{v_{T\alpha}^2}[1-\sqrt{\pi}b_\alpha\exp(b_\alpha^2)\{(1-\mathrm{erf}(b_\alpha)\}]\equiv\frac{2n_{\alpha 0}}{v_{T\alpha}^2}F_\alpha, \tag{19}$$

where $b_\alpha=\gamma/kv_{T\alpha}$ and F_α is short hand for the expression between square brackets. Combining this with Eqs. (7)–(9) and (11), we obtain the general dispersion law as

$$\left(1+\frac{F_e}{k^2\lambda_{De}^2}+\frac{F_i}{k^2\lambda_{Di}^2}\right)\left(1-\frac{F_d}{k^2\lambda_{Jd}^2}\right)+\frac{F_d}{k^2\lambda_{Dd}^2}=0. \tag{20}$$

To understand modifications of dispersion properties due to the kinetic description, let us start from the dust Langmuir waves and consider Eq. (20) in the case $k^2\lambda_D^2 \gg 1$. Relation (20) is plotted in Fig. 1 for the dimensionless variables $y=\omega^2/(\omega_{Jd}^2-\omega_{pd}^2)$ and $x=k^2/k_{cr}^2$, along with the dispersion relation for the fluid model, which corresponds to $y=-1+x$. As could be expected, the perturbations indeed grow, if $k<k_{cr}$, although the growth rates generally are quite different in both plasma models. This result is consistent with the kinetic analysis of a self-gravitating neutral system [17], since the Jeans frequency enters the equations as part of the effective Jeans frequency through $\omega_{J,\mathrm{eff}}^2=\omega_{Jd}^2-\omega_{pd}^2$.

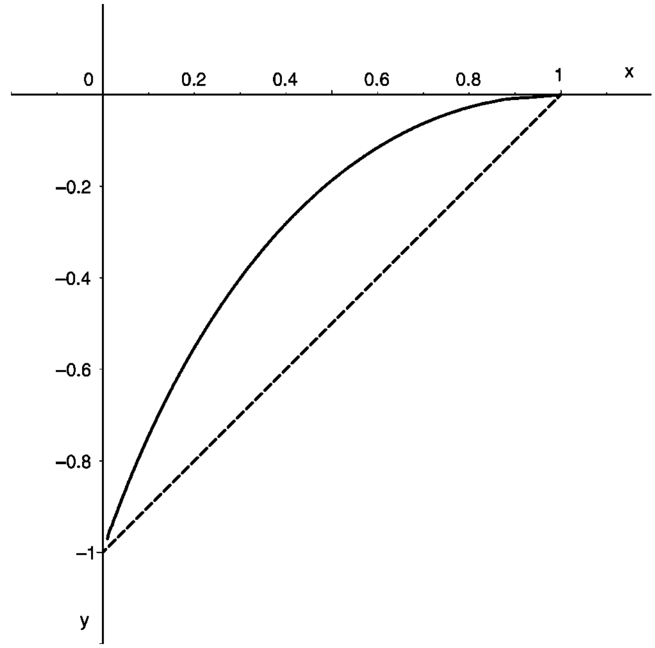

FIG. 1. Kinetic dispersion relation (20) (full curve) and fluid dispersion (dotted curve) in the case $k^2\lambda_D^2 \gg 1$. Only the unstable branch is plotted.

Next we consider the approximate solution of Eq. (20) in the regime of the dust-acoustic waves ($k_{cr}^2\lambda_D^2 \ll 1$), when $k\to k_{cr}-0$. Introducing dimensionless variables $y=\omega^2/\omega_{Jd}^2$ and $x=k^2/k_{cr}^2$, we can expand the functions $F_\alpha(\sqrt{-y/2x})$ for small arguments, i.e., $y/x \ll 1$. Then one can obtain the simplified dispersion relation as $y=-x(1-x)^2/\pi$. Figure 2 demonstrates the difference in the growth rates of the dust-acoustic perturbations in the kinetic and fluid approaches in the vicinity of $k\to k_{cr}-0$.

Thus, although both the fluid and kinetic models are unstable if $k<k_{cr}$, the growth rates are quite different: the perturbations in the fluid description grow faster than in the kinetic one.

III. ELECTROSTATIC WAVES IN SELF-GRAVITATING PLASMAS

Here we analyze the stable modes in a self-gravitating plasma when $k>k_{cr}$. To evaluate the integral (10) for the Maxwellian distribution (12), we must compute the integral over v according to the well-known ''Landau bypass rule'': the contour of integration in the complex v plane bypasses the pole singularity at $\omega=kv$ from below. Then one can get

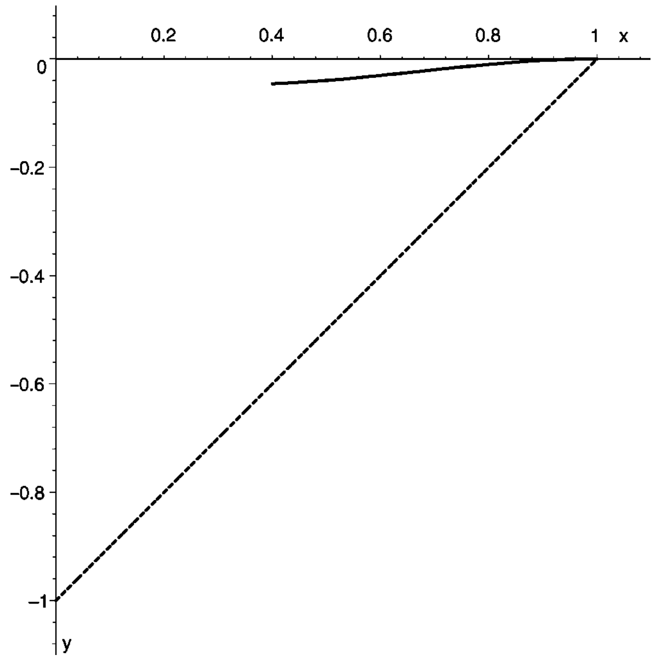

FIG. 2. As in Fig. 1, but for $k^2\lambda_D^2 \ll 1$.

$$I_\alpha = \frac{2n_{\alpha 0}}{v_{T\alpha}^2}[1 + i\sqrt{\pi} z_\alpha W(z_\alpha)], \tag{21}$$

where $z_\alpha = \omega/kv_{T\alpha}$ is a dimensionless frequency and W is the Kramp function [22]

$$W(z) = \exp(-z^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^z \exp x^2 dx\right]. \tag{22}$$

Note that another function is often used, which is related to Eq. (22) by $Z(z) = i\sqrt{\pi} z W(z)$ and is tabulated [23].

The final dispersion equation (11) can be written in the form

$$\varepsilon(\omega,k) = 1 + \sum_\alpha \frac{1 + i\sqrt{\pi} z_\alpha W(z_\alpha)}{k^2\lambda_{D\alpha}^2} + \frac{\left[\sum_\alpha \frac{1 + i\sqrt{\pi} z_\alpha W(z_\alpha)}{k^2\lambda_{D\alpha}\lambda_{J\alpha}}\right]^2}{1 - \sum_\alpha \frac{1 + i\sqrt{\pi} z_\alpha W(z_\alpha)}{k^2\lambda_{J\alpha}^2}} = 0, \tag{23}$$

which we analyze below in different frequency regimes.

The analytic analysis of plasma mode propagation requires asymptotic expansions of the dispersion function $W(z)$ for either small or large arguments. We will use the following approximations [22]:

(a) $|z| \gg 1$, $\mathrm{Re}\{z\} \gg \mathrm{Im}\{z\}$, $\mathrm{Im}\{z\} < 0$,

$$W(z) = \frac{i}{\sqrt{\pi} z}\left(1 + \frac{1}{2z^2} + \frac{3}{4z^4} + \dots\right) + \exp(-z^2) \tag{24}$$

and

(b) $|z| \ll 1$,

$$W(z) = 1 + \frac{2iz}{\sqrt{\pi}} + \cdots. \tag{25}$$

The wave frequency ω and dispersion relation $\varepsilon(\omega,k)$ are then written in terms of real and imaginary parts $\omega = \omega_0 + i\gamma$ and $\varepsilon(\omega,k) = \varepsilon_r + i\varepsilon_i$, where we assume that $|\omega_0| \gg |\gamma|$ and $|\varepsilon_r| \gg |\varepsilon_i|$.

We consider the low-frequency case, when the wave frequency satisfies the inequality $kv_{Td} \ll \omega \ll kv_{Ti}, kv_{Te}$ and the dust particles are involved in the wave processes. Some qualitatively different solutions are possible, depending on the relative magnitude of ω_{pd}^2, ω_{Jd}^2, and $k^2\lambda_D^2$. To discuss the low-frequency waves in such plasmas, we introduce a parameter

$$\Delta = \frac{\omega_{Jd}^2}{\omega_{pd}^2}\left(1 + \frac{1}{k^2\lambda_D^2}\right), \tag{26}$$

which measures the influence of self-gravitation in a medium. As can be seen, the boundary between stable and unstable solutions occurs in the vicinity of $\Delta = 1$, when the gravitational and electric interaction forces balance. The dispersion relation (23), still assuming $\omega \gg kv_{Td}$, is now of the form

$$1 + \frac{1}{k^2\lambda_D^2}\left[1 + \frac{i\omega\sqrt{\pi}}{k(1+\delta)}\left(\frac{1}{v_{Ti}} + \frac{\delta}{v_{Te}}\right)\left(1 + \frac{\omega_{Jd}^2}{\omega^2}\right)\right] - (1-\Delta) \times \left[\frac{\omega_{pd}^2}{\omega^2}\left(1 + \frac{3k^2v_{Td}^2}{2\omega^2}\right) - i\sqrt{\pi}\,\frac{\omega}{kv_{Td}}\frac{1}{k^2\lambda_{Dd}^2} \times \exp\left(-\frac{\omega^2}{k^2v_{Td}^2}\right)\right] = 0, \tag{27}$$

where a parameter $\delta = \lambda_{Di}^2/\lambda_{De}^2$ measures the influence of the electron component on the dust modes. If the dust grains are negatively charged, plasma electrons are absorbed by the charged dust and hence n_{e0} decreases, implying a decrease of δ to a value $\delta < 1$ or even to $\delta \ll 1$. The latter corresponds to a situation where almost all plasma electrons have been absorbed by the dust grains, and this simplified model of a dusty plasma merely consisting of ions and charged grains is known and discussed in the literature [24]. The opposite occurs in the specific case of positively charged grains in an isothermal plasma ($T_e \simeq T_i$), giving $\delta > 1$. The general dispersion relation (27) can be adapted to both cases.

As the imaginary part of Eq. (27) is small compared to the real part, we can easily apply the Taylor expansion of the dispersion relation around $\gamma = 0$, which yields

$$\omega_0^2=\frac{\omega_{pd}^2(1-\Delta)}{1+\frac{1}{k^2\lambda_D^2}}\left[1+\frac{3k^2\lambda_{Dd}^2\left(1+\frac{1}{k^2\lambda_D^2}\right)}{1-\Delta}\right] \tag{28}$$

and

$$\gamma=-\varepsilon_i(\omega_0)\left[\frac{\partial\varepsilon_r}{\partial\omega}\bigg|_{\omega=\omega_0}\right]^{-1}$$
$$=-\frac{\sqrt{\pi}\omega_0^4}{2k^3\lambda_D^2\omega_{pd}^2(1-\Delta)}\left[\frac{\frac{1}{v_{Ti}}+\frac{\delta}{v_{Te}}}{(1-\Delta)(1+\delta)}+\frac{(1-\Delta)\beta}{v_{Td}}\right.$$
$$\left.\times\exp\left(-\frac{\omega_0^2}{k^2v_{Td}^2}\right)\right], \tag{29}$$

where the parameter $\beta=\lambda_D^2/\lambda_{Dd}^2$ is introduced. It follows immediately that the above mentioned condition $\omega\gg kv_{Td}$ is satisfied only if

$$k^2\lambda_{Dd}^2+\frac{1}{\beta}\ll|1-\Delta|. \tag{30}$$

This means that weakly damped low-frequency modes with frequencies given through Eq. (28) can only exist in the long-wavelength range ($k^2\lambda_{Dd}^2\ll|1-\Delta|$) and in dusty plasmas for which $1\ll\beta|1-\Delta|$. Compared to the conditions for low-frequency modes in the usual dusty plasmas, both of these inequalities become stricter as self-gravitation is taken into account. As for the wavelength of the low-frequency perturbations, it can be either shorter or larger than the plasma Debye length.

A. Analog of dust-acoustic modes

In the long-wavelength limit, when $k^2\lambda_D^2\ll 1$, the dispersion law (28) corresponds to the analog of the dust-acoustic mode in dusty self-gravitating plasmas. The resulting frequency and damping decrement are given by

$$\omega_0^2\simeq k^2v_{da}^2\left[(1-\Delta)(1-k^2\lambda_D^2)+\frac{3}{\beta}\right] \tag{31}$$

and

$$\gamma=-\sqrt{\frac{\pi}{8}}kv_{da}\left\{\left(\frac{\omega_{pd}}{\omega_{pi}}+\frac{\omega_{pd}}{\omega_{pe}}\delta^{3/2}\right)(1+\delta)^{-3/2}\right.$$
$$\left.+(1-\Delta)^2\beta^{3/2}\exp\left[-\frac{3+(1-\Delta)\beta}{2}\right]\right\}. \tag{32}$$

Here the parameter Δ has been redefined as $\Delta\simeq\omega_{Jd}^2/k^2v_{da}^2$. In the absence of self-gravitation ($\Delta=0$), Eqs. (31) and (32) simply reduce to the dispersion relations obtained in the kinetic approach for dust-acoustic waves, and thus Eqs. (31) and (32) in some sense generalize earlier results [14].

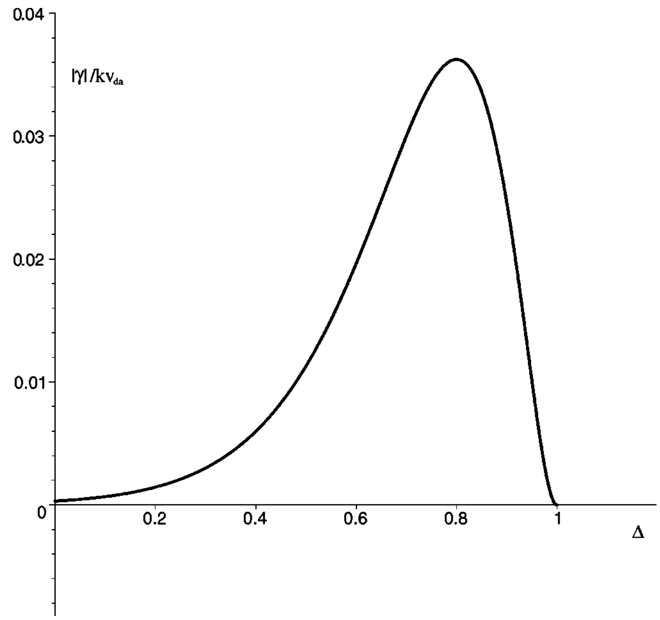

FIG. 3. Effect of self-gravitation on Landau damping of the dust-acoustic wave for $\beta=20$.

Assuming $\Delta<1$, we consider the influence of self-gravitation on the dust-acoustic modes. First of all, we should note that dust-acoustic waves usually are almost non-dispersive in the long-wavelength regime ($k^2\lambda_D^2\ll 1$). Indeed, the phase velocity of long-wavelength disturbances is $v_{ph}=\omega_{pd}\lambda_D$. In contrast to that, in a self-gravitating plasma $v_{ph}=\omega_{pd}\lambda_D(1-\omega_{Jd}^2/k^2v_{da}^2)^{1/2}$, and this mode demonstrates the dispersion, which is completely specified by the self-gravitational effects. Furthermore, a peculiarity of the dust-acoustic wave in a self-gravitating plasma is its damping [Eq. (32)]. Since the dust species are sufficiently heavy, it is reasonable to assume that the second term in the curly brackets of Eq. (32) makes the main contribution to the damping rate, and the latter is almost completely controlled by self-gravitation. But this has to be treated with some care, when dealing with large values of the parameter β. For very large β, the exponential term in Eq. (32) is reduced and consequently, the influence of self-gravitation on the damping rate diminishes. Thus, when studying self-gravitational effects we have to restrict our analysis to those values of β for which the second term of Eq. (32) prevails. This is equivalent to saying that inside the curly brackets the term in $\beta^{3/2}\exp(-\beta/2)$ contributes significantly compared to $\omega_{pd}/\omega_{pi}+\omega_{pd}\delta^{3/2}/\omega_{pe}$. In addition, ω_{pd} should exceed ω_{Jd} but not to a great extent, lest the self-gravitational effects again become insignificant. With regard to the possible parameters of self-gravitating plasmas, this leads quite realistically to assume that β remains smaller than 50.

Figure 3 illustrates the typical evolution of the wave decrement in such self-gravitational plasmas: the ratio $|\gamma|/kv_{da}$ for $\beta=20$ is shown as a function of Δ. The decrement rate for a usual dust-acoustic wave in dusty plasmas without self-gravitation corresponds to a value $|\gamma|/kv_{da}$ for $\Delta=0$. As one

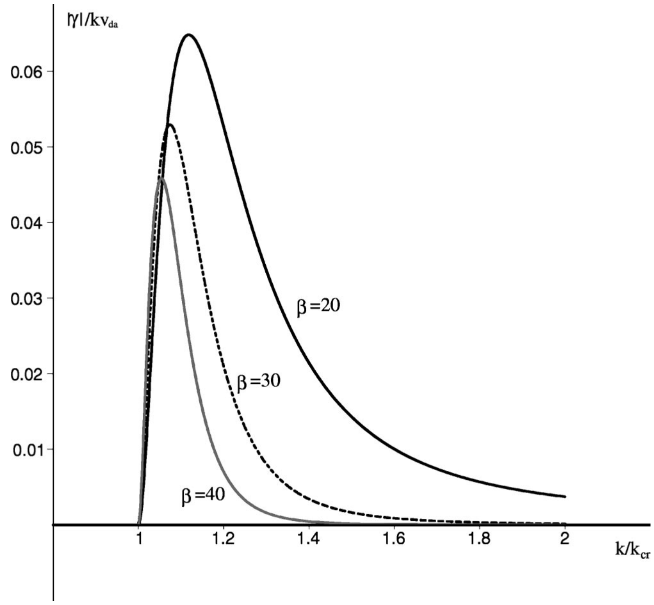

FIG. 4. Landau damping rate vs k/k_{cr} for the analog of the dust-acoustic wave in self-gravitating plasmas.

can see from Fig. 3, the curve demonstrates a considerable growth of the damping effect, and the influence of self-gravitation increases especially in the vicinity of $\Delta\to 1$. Then the damping drops to zero for $\Delta=1$, when $\omega_{Jd}^2=k^2v_{da}^2$.

From a physical point of view, the explanation of the rather high damping rate for $\Delta\to 1$ is quite simple. The damping rate of a wave is proportional to the difference in numbers of slow and fast captured particles, which is determined by $-(df_{d0}/dv)_{v=\omega_0/k}$. This means that in the case of a Maxwellian distribution (12) the number of dust particles that can effectively interact with the dust-acoustic wave is given by

$$N\sim(1-\Delta)^{1/2}\exp\left[-\frac{1}{2}(1-\Delta)\beta\right]. \qquad (33)$$

When the self-gravitational influence increases ($\Delta\to 1$), the number of resonant dust particles N increases also, achieving a maximum at $\Delta_m=1-1/\beta$. Hence in the vicinity of $\Delta\to 1$ the number of resonant particles becomes so large that the damping rate grows crucially, far exceeding the Landau damping rate for usual dust-acoustic waves. Note also that the picture of absorption of the dust-acoustic waves by resonant dust particles refers to the case when Eq. (30) is satisfied. This means that the current considerations are not valid in the immediate vicinity of $\Delta\sim\Delta_m$ and only provide a physical idea of the anomalous damping rate due to self-gravitational effects.

The appearance of a large damping rate with increasing Δ may be interpreted in a different way. Pursuing the analogy with the fluid model, we use the critical Jeans wave number (16), which can be estimated as $k_{cr}\simeq\omega_{Jd}/v_{da}$, and consider the dimensionless damping rate $|\gamma|/\omega_{Jd}$ defined by Eq. (32) as a function of k/k_{cr} in the range $k/k_{cr}>1$. The corresponding dependence is shown in Fig. 4 for different β. As expected, the perturbations damp strongly when $k\to k_{cr}+0$. Clearly, a region of strong damping is determined by the parameters of the plasma, in particular by β. With an increase of β, the influence of self-gravitation is reduced and the curve approaches the usual damping rate for dust-acoustic waves.

Thus, in contrast to the damping rate of a usual dust-acoustic wave, for which the damping rate is a linear function of the wave number, i.e., $|\gamma|\sim kv_{da}$, the latter can be modified considerably in a self-gravitating plasma. In particular, if the value β is typically no more than 50, there always exist disturbances with wave numbers $k\sim\omega_{Jd}(1+1/2\beta)/v_{da}$, which are subjected to strong attenuation due to self-gravitational effects. This means that in a medium with specified values of the dust plasma (ω_{pd}) and Jeans (ω_{Jd}) frequencies and $1<\beta<50$, dust-acoustic waves with $k\sim\omega_{Jd}(1+1/2\beta)/v_{da}$ can hardly propagate.

Therefore the resemblance of the fluid and the kinetic model of dust sound waves in dusty self-gravitating plasmas breaks down at larger wave numbers $k>k_{cr}$. Here the fluid dispersion equation supports modes that can simply be regarded as undamped gravity-modified dust-acoustic waves. The kinetic analysis of self-gravitating plasmas is more complicated. We found that all dust-acoustic perturbations in self-gravitating plasmas with wave numbers $k>k_{cr}$ are damped due to the collisionless Landau damping, particularly in the vicinity of $k\to k_{cr}+0$.

B. Dust Langmuir waves

For sufficiently short wavelengths, when $k^2\lambda_D^2\gg 1$ but $k^2\lambda_{Dd}^2\ll|1-\Delta|$, the spectrum is given by

$$\omega_0^2\simeq\omega_{pd}^2(1-\Delta+3k^2\lambda_{Dd}^2), \qquad (34)$$

where the parameter Δ is defined as $\Delta=\omega_{Jd}^2/\omega_{pd}^2$. If $\Delta<1$ this mode is attenuated, with damping rate

$$\gamma\simeq-\sqrt{\frac{\pi}{8}}\frac{\omega_{pd}}{k^3\lambda_D^3}\left\{\left(\frac{\omega_{pd}}{\omega_{pi}}+\frac{\omega_{pd}}{\omega_{pe}}\delta^{3/2}\right)(1+\delta)^{-3/2}+(1-\Delta)^2\beta^{3/2}\exp\left[-\frac{3}{2}-\frac{(1-\Delta)\omega_{pd}^2}{k^2v_{Td}^2}\right]\right\}. \qquad (35)$$

Equations (34) and (35) reduce to the dispersion relations for the usual Langmuir modes (e.g. ion Langmuir waves in an electron-ion plasma), however, the role of the plasma frequency is played by the effective plasma frequency of the dust particles defined through $\omega_{p,\mathrm{eff}}^2=\omega_{pd}^2-\omega_{Jd}^2$. The spectrum of plasma waves is quite well known [12,13] and here we shall not discuss it further. We just note that disturbances of that mode can be subjected to the same strong damping as dust-acoustic modes in self-gravitating plasmas if $\Delta\to 1$, i.e., $\omega_{pd}\to\omega_{Jd}$.

IV. RESULTS

To summarize, the stability of low-frequency waves in homogeneous self-gravitating plasmas in a kinetic description is closely related to the stability of the analogous hydrodynamic model: in both cases there is instability if and only if the wave number of the disturbances is less than a critical value. However, the growth rates of the Jeans instability are quite different: the perturbations in a fluid description grow faster than in a kinetic one. The stable solutions of the two models are quite different also: the fluid model supports undamped short-wavelength dust-acoustic waves, but the kinetic model can crucially alter the conditions for the propagation of the dust-acoustic waves. It is found that all stable dust-acoustic perturbations in self-gravitating plasmas are damped due to the collisionless Landau damping. The latter strongly depends on the values of the plasma parameters, especially on β. If this is large enough ($\beta > 50$), the damping rate differs only slightly from the damping rate for the usual dust-acoustic waves. In contrast to this, a self-gravitating plasma with $1 < \beta < 50$ shows a considerable growth of the damping effect, particularly for wave numbers near the critical values. This means that in such a self-gravitating plasma there exists a range of wave numbers (or equivalently, wavelengths), where dust-acoustic modes can hardly be excited.

ACKNOWLEDGMENTS

This work was supported by the Bijzonder Onderzoeksfonds of the RUG through research (F.V. and G.J.) and foreign visitor (V.V.Y.) grants.

[1] N.N. Rao, P.K. Shukla, and M.Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[2] A. Barkan, R.L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[3] C. Thompson, A. Barkan, N. D'Angelo, and R.L. Merlino, Phys. Plasmas **4**, 2331 (1997).
[4] K. Avinash and P.K. Shukla, Phys. Lett. A **189**, 470 (1994).
[5] B.P. Pandey, K. Avinash, and C.B. Dwivedi, Phys. Rev. E **49**, 5599 (1994).
[6] P. Meuris, F. Verheest, and G.S. Lakhina, Planet. Space Sci. **45**, 449 (1997).
[7] F. Verheest, P.K. Shukla, N.N. Rao, and P. Meuris, J. Plasma Phys. **58**, 163 (1997).
[8] B.P. Pandey and G.S. Lakhina, Pramana **50**, 191 (1998).
[9] F. Verheest, M.A. Hellberg, and R.L. Mace, Phys. Plasmas **6**, 279 (1999).
[10] P. Bliokh, V. Sinitsin, and V. Yaroshenko, *Dusty and Self-Gravitational Plasmas in Space* (Kluwer, Dordrecht, 1995).
[11] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, 2000).
[12] N.A. Krall and A.W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).
[13] A.I. Akhiezer, I.A. Akhiezer, R.V. Polovin, A.G. Sitenko, and K.N. Stepanov, *Plasma Electrodynamics* (Pergamon Press, Oxford, 1975).
[14] F. Melandsø, T.K. Aslaksen, and O. Havnes, J. Geophys. Res. [Space Phys.] **98**, 13 315 (1993).
[15] A. Brattli, O. Havnes, and F. Melandsø, J. Plasma Phys. **58**, 691 (1997).
[16] A.M. Fridman and V.L. Polyachenko, *Physics of Gravitating Systems. Equilibrium and Stability* (Springer-Verlag, Berlin, 1984).
[17] J. Binney and S. Tremaine, *Galactic Dynamics* (Princeton University Press, Princeton, 1994).
[18] F. Melandsø, T. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).
[19] R.K. Varma, P.K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[20] N.N. Rao and F. Verheest, Phys. Lett. A **268**, 390 (2000).
[21] P.V. Bliokh and V.V. Yaroshenko, Astron. Zh. **62**, 569 (1985). [Sov. Astron. **29**, 330 (1985)].
[22] V.N. Fadeeva and N.M. Terentjev, *Tablitsy Znacheniy Integrala Veroyatnosty of Kompleksnogo Argumenta* (GITTL, Moscow, 1954).
[23] B.D. Fried and S.D. Conte, *The Plasma Dispersion Function* (Academic Press, San Diego, 1961).
[24] C.K. Goertz, Rev. Geophys. **27**, 271 (1989).